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# DIDACTIC ENGINEERING IN THE DEVELOPMENT OF NUMERICAL THINKING

## INGENIERÍA DIDÁCTICA EN EL DESARROLLO DEL PENSAMIENTO NUMÉRICO

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### RESUMEN.

El desarrollo del pensamiento numérico en la enseñanza primaria es un aspecto importante de la formación matemática de los niños, ya que permite una comprensión general de números y operaciones para facilitar el desarrollo de su manejo y aplicación. La investigación aplica una estrategia pedagógica basada en la ingeniería

didáctica, que permite a los alumnos identificar y formular preguntas relacionadas con el manejo y la comprensión de los números y sus operaciones básicas, junto con la decisión de abordar individualmente y de manera grupal el planificar y tomar decisiones. El estudio se enmarca en el paradigma cuantitativo, en un diseño cuasi-experimental, trabajando con grupos control y experimental. El pre-test se aplica para identificar el desarrollo del pensamiento numérico en los estudiantes y como apoyo al diseño de la propuesta de intervención. Una vez elaborada la propuesta, se evalúa mediante un post-test que permite identificar mejoras significativas en el pensamiento numérico de los niños del grupo experimental en comparación con los del grupo de control. Se concluye que las estrategias

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implementadas desde la ingeniería didáctica representan una alternativa para el desarrollo de temas asociados al fortalecimiento del pensamiento numérico em la niñez.

## ABSTRACT.

The development of numerical thinking in primary education is an important aspect of children's mathematical training, since it allows for a general understanding of numbers and operations in order to facilitate the development of their handling and application. The work allows the implementation of a pedagogical strategy based on didactic engineering, which allows students to identify and formulate questions related to the management and understanding of numbers and their basic operations together with the decision to approach individually and which ones in a group way while planning and making decisions in their development and evaluation of results. The study is framed in the quantitative paradigm, in a quasi-experimental design, working with control and experimental groups. Pre-test is applied to identify the development of numerical thinking in students and as a support to the design of the intervention proposal. Once the proposal is developed, it is evaluated by means of a post-test that allows identifying significant improvements in the numerical thinking of the children of the experimental group in comparison with those of the control group. This leads to the conclusion that the developed intervention contributes favorably to the development of numerical thinking in children.

## 1. INTRODUCTION

The new approaches that are emerging in the education sciences not only facilitate the appropriation of knowledge itself as an end in itself, but also the need to form an autonomous subject that can successfully face the demands of the citizen. It is no longer a question of students learning, but rather of being able to manage their

own learning in an intelligent and autonomous manner, and that the knowledge they acquire be useful in daily life for solving problems and dilemmas of all kinds, from those of a practical and domestic order to the most profound ethical and social ones [1].

Today's education has a new and greater responsibility: instead of offering information, it must train and enable the learner to seek it, select it critically, understand it and apply it effectively to his or her personal needs and those of society. This leads to a lot of changes, and even more, in the way mathematics teaching is conceived at all levels of education, but mainly at the elementary level. From this point of view, it can be stated that mathematics, more than an area of study, constitutes a tool that helps to understand the surrounding reality, in which understanding the magnitude of its contributions in other disciplines and in everyday life, requires a change in the conception of mathematics [2], due to the fact that it has become established in people's thinking as a complex discipline that is difficult to approach. Therefore, it is necessary to build new strategies for its permanent teaching, which are in accordance and coherent with the constant situation of experimentation and change in which we live [3].

In this respect, children develop conceptualizations about the writing of numbers, based on the information they extract from spoken numbering and on their knowledge of conventional writing [4]. This has implications for writing, since when the child communicates orally to express numerical aspects, he or she does so spontaneously, but when trying to express himself or herself in writing regarding numbering, he or she must take into account specific rules such as the positional value of numbers, additive character, and correspondence, among others. In such a way that, in the didactic processes it is necessary to help the students in the processes of construction of the concept of number.



Therefore, teachers have the challenge of re-signifying their pedagogical practices so that their students can appropriate the concepts and understand the importance of mathematics. It is then necessary to look for alternative ways of presenting the content based on situations and activities that represent a significant meaning for the student; these will allow the students to generate conjectures, analyze them with their classmates and apply the knowledge acquired previously [5]. In this sense, what is fundamental is not the way in which the teacher thinks, since it is not a question of saying, but of believing that the participation of students is a valuable element of the formative process. Similarly, it is noted that teachers must give space to the unbalanced dialogue based on questions and concerns [6]. From the constructivist paradigm, students are not depositaries of knowledge, so they can put forward their positions, ideas and thoughts regarding a topic. Moreover, Vergel et al [7] favours group work, which allows students the opportunity to carry out, in a more successful way, more complex skills than what they can do by themselves. Therefore, some premises of the constructivist approach can be synthesized: the teacher appears as a mediator, who allows the child greater participation, given that knowledge is acquired through a process of construction; for learning, social interaction is put into play from which children assimilate, organize and adapt the objects of knowledge; previous experiences and knowledge constitute essential aspects in learning, for which reason teaching must allow new knowledge to be organized on the basis of previously acquired knowledge [8]. It is therefore necessary to propose alternative solutions for educational practice, among which is the suggestion and exercise of new teaching strategies that facilitate the development of competencies and skills in all areas, including mathematics. The strengthening of competencies associated with the area would have an impact in the medium term on the results of standardized tests for measuring the quality of education and

student performance. Consequently, the general purpose of this paper is to propose the use of a set of didactic strategies based on didactic engineering aimed at improving the mathematical thinking of students in basic education.

## 2. METHOD

The research is framed within the quantitative paradigm with both descriptive and inferential approaches [9,10]. The following variables are identified: dependent variable: level of appropriation and understanding that the students in the sample have with respect to the standards associated with numerical thinking. Independent variable: The didactic sequences framed within the methodology called didactic engineering, which are based on the theories of didactic situations and semiotic representations [11]. The research hypothesis suggests that with the implementation of the sequences if there are improvements in the academic performance of students [12].

Procedure carried out. The research process was based on the methodology called didactic engineering, which defines four stages that were mentioned very briefly. Preliminary Analysis Stage [13]. This corresponds to the collection of data to establish the origin of the problem in order to facilitate its understanding. It analyzes the epistemological origin of the concept, and then reviews the characteristics of the dynamics of teaching it in the classroom and the concepts that derive from such teaching [14]. This stage was consolidated with the application of the knowledge test to the students in the sample. Analysis stage to Priori. This phase describes and predicts what the student will do in the experiment [15]. These predictions do not come out of the blue, they are based on some pedagogical current, for example: theory of didactic situations, constructivist theory, theory of semiotic representations, critical mathematical theory, among others; with the purpose of



generating Didactic Sequences associated to the subject under study, with which it is intended to control the students' behaviours and try to solve situations of conceptual or procedural difficulty [16]. This stage is evidenced in the research in the process of design and validation of the three didactic sequences created.

Experimental stage. It is associated with the field work, since it is in this stage of the process where there is contact with the student and his or her conceptions. It is important at this stage that the researcher not only limits himself to the answers provided by the student, but also identifies those aspects that present greater difficulty, the characteristics of the doubts and other aspects that may tend to improve the teaching process. This phase was completed in the course of three weeks, a period of time in which the didactic sequences were applied.

Evaluation and Post-Assessment Stage. It is based on the set of data collected throughout the experimentation such as observations made of the teaching sequences, student productions inside or outside the classroom, among others; to finally compare the results of the two analyses, the a priori and posteriori. This phase is carried out when the aim is to determine the effect of the implementation of the teaching sequences, which is supported by a descriptive exploration of the results and then to ratify the conclusions through the validation of the hypothesis system.

Instruments used. Two types of instruments were designed and implemented for data collection: Knowledge test and Didactic engineering. In Knowledge test [17], this instrument is applied in two stages of research and aims to determine the level of mastery that students have of mathematical skills associated with numerical thinking. The test is composed of several items, which are mainly focused on the resolution of everyday situations expressed in natural language [18], which in some cases is supported by a graphic, tabular or bar chart representation,

to later give answers to a series of questions oriented to the development of communication, reasoning and problem solving processes [19]. The selection of situations is based on various criteria defined in the so-called reference matrix, a document issued by the Ministry of National Education, which aims to provide guidance to teachers on their work in the classroom.

Didactic engineering. To the pedagogical model of the school [20], which is the one developed at the site where the research was conducted, a modification process was carried out in which three didactic sequences were designed, which were based on Brousseau's theory of didactic situations [21] and on Duval's theory of semiotic representations [22] for the design of the different activities. The dynamics of the application of the sequences incorporates a phase of cooperative work in which the student is expected to formulate all his doubts and initially [23], these are attended to with his team until he assumes a position as a group, which at the moment of the plenary is agreed upon with the whole class and thus tries to build up the classroom knowledge.

### 3. RESULTS

In order to evaluate the levels of development in each of the competencies, a test was applied with equal difficulty in two moments: a pre-test at the beginning of the investigation and a post-test at the end to also evaluate the results of the intervention. Each test is composed of twelve questions distributed in such a way that they can evaluate the three defined competencies of interest for the research. In each competence (Figura 1), the learning to be achieved and the difficulty found in each of the evidences that constitute it are identified. The two percentages represent the success rates in the control and experimental groups, respectively. The results obtained in the pretest: reasoning and argumentation competence (37%, 43%); communication, representation and



modelling competence (44%, 36%); problem-solving competence (70%, 73%). The pretest post-test contrast in the experimental group  $z=-2.809d$  with bilateral asymptotic significance at 99% in each category, i.e. there are significant differences in numerical thinking skills, the Wilcoxon range test in subcategories, for which there are similarly significant contrasts in each of them at 95% with  $z=-2.53d$  for argumentation,  $z=-2.170$  for communication and modelling,  $-2.131$  for representation and problem solved

(Figure 1), i.e. similar results are observed in the two groups; in both, higher scores are obtained in the competence of posing and solving problems; therefore, it is possible to infer that the competences that can most affect the development of numerical thinking are those related to reasoning, argumentation, communication and representation. This suggests that the intervention plan should be designed to strengthen these skills, without neglecting problem solving, developing it in correspondence with the previous ones.

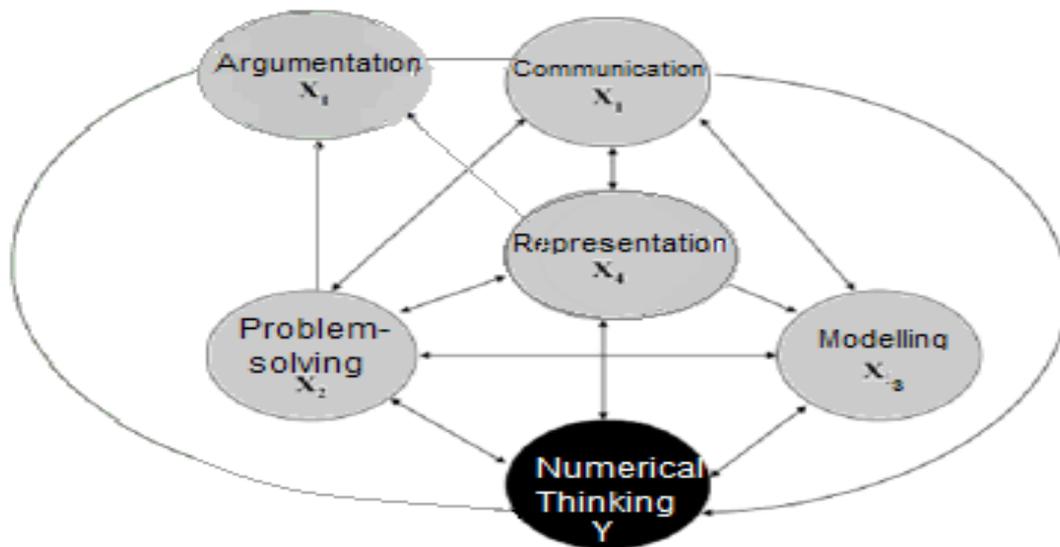


Figura 1. Competences. Fuente: Autores

The Mann-Whitney U test with a mean range of 88.5 for the control group (60% successful response), and a mean range of 124 for the experimental group (85% successful response), shows affirmative bilateral asymptotic significance, i.e. the strategy influences and the greater the range of the experimental group means that there was improvement. The intervention plan was designed based on the results of the pre-test and had the contribution of the children who participated in conversations and reflections on issues to be developed and their purpose. The design of each classroom pedagogical project includes theme, execution

time, objective, didactic situations, activities, means and resources, and evaluation. The children actively participate in the development of each project, identify and decide on activities to be carried out individually and those to be carried out in groups, develop them, present a report and finally evaluate their work.

The projects designed and carried out in the pedagogical intervention are the following Adding, adding, adding; let's find the difference; learning to multiply by playing; the world of distributions; having fun with mathematical problems; fractioning quantities. Each project is supported by a student work guide. The



following results were obtained in the post-test: reasoning and argumentation competence (42%, 83%); communication, representation and modelling competence (45%, 56%); problem-solving and posing competence (75%, 93%). The pedagogical intervention improves the results in the competence assessment test, which in turn indicates that it provides a basis for the development of numerical thinking in third grade students.

#### 4. CONCLUSIONS

The pedagogical intervention aimed at developing mathematical skills associated to numerical thinking consisted in the construction of three didactic sequences which were coherently articulated with the mathematical processes and concepts of numerical thinking for this grade. In the realization of these sequences, several representation records were incorporated with the intention of potentializing the interpretation of data, and then suggesting a series of questions that demanded from students the application of reasoning, transformations between representation records, the calculation of probabilities, as well as the use of diverse counting techniques to obtain results and the contextualization of them in the light of the proposed situations.

Mathematics is considered to be one of the central contents of primary school, and for this reason one should take advantage of all the circumstances that allow one to consolidate the knowledge that one is trying to teach, beyond the moments established within each classroom to work with mathematics. In this sense, the entire empirical study began with the objective of diagnosing the dimensions of numerical thinking in third grade students, prior to the implementation of the proposal.

The design of the proposal was made in such a way that it allowed the children to interact

with the most difficult topics in an enjoyable way and with useful exercises for life. In this way, it is possible to reflect on the importance of making the pedagogical task of mathematics attractive, for which a permanent search for new challenges must be started, where theory can be integrated with practice, and where each teacher must look for the possibility of giving and receiving, of learning from themselves and from others; that is, to give meaning to the planning of pedagogical practices.

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